Beyond Points and Beams: Higher-Dimensional Photon Samples for Volumetric Light Transport Supplemental Document

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1 CONTROL VARIATES

Estimating the contribution of a blurred photon plane involves the computation of an integral along the interval of overlap between the camera segment and the photon plane. In order to reduce variance introduced by this estimation, we apply *control variates* [Glasserman 2003] in our implementation of the photon plane estimator for homogeneous media.

The concept of control variates is simple. Say we wanted to estimate the integral of some function $f(x)$ over a domain Ω using Monte Carlo integration:

$$\int_\Omega f(x)\,\mathrm{d}x \approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)}. \tag{1}$$

Integrating $f(x)$ analytically may not be feasible, but say we had access to some other function $g(x)$ whose integral $G(\Omega)$ over the domain is known. $g(x)$ is then referred to as the *control variate*, and we can form a new estimator of the integral of $f(x)$ with

$$\int_\Omega f(x)\,\mathrm{d}x \approx G(\Omega) + \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)-g(x_i)}{p(x_i)}. \tag{2}$$

If $g(x)$ is correlated with the integrand $f(x)$, then the Monte Carlo estimator in Eq. (2) has less variance compared to the straightforward MC estimator.

We repeat the contribution of a blurred photon plane, given by Eq. (17) in the main paper:

$$\langle D\rangle^{l-1,k}_{\text{Q-B1D}} = \int_{s_{k-}}^{s_{k+}} f(\tilde{t}_{l-1})f(\tilde{t}_l)\left\{\frac{K_1(\mathbf{x}_l,\tilde{\mathbf{y}}_k)}{J^{l-1,l}_{\text{Q-B1D}}}f^{l,k}_\omega\right\} f(s)\,\mathrm{d}s. \tag{3}$$

Evaluating this expression involves computing an integral over the segment of overlap $[s_{k-}, s_{k+}]$ between the camera ray and the photon plane.

Because the directionality of the blur is orthogonal to the photon plane, the distance throughputs $f(\tilde{t}_{l-1})$ and $f(\tilde{t}_l)$ implicitly depend on the integration variable s, which complicates analysis slightly. We note that the blur direction of these two terms is not of major practical concern, and as a first simplification we replace $\tilde{t}_{l-1}$ and $\tilde{t}_l$ by their values at the center of the interval of overlap. This changes the directionality of the blurring, so that these quantities are blurred along the camera segment instead of the plane normal.

This allows us to move the majority of terms out of the integrand to obtain

$$\langle D\rangle^{l-1,k}_{\text{Q-B1D}} = C\int_{s_{k-}}^{s_{k+}} f(s)\,\mathrm{d}s, \tag{4}$$

$$\text{with} \qquad C = f(t_{l-1})f(t_l)\left\{\frac{K_1(\mathbf{x}_l,\mathbf{y}_k)}{J^{l-1,l}_{\text{Q-B1D}}}f^{l,k}_\omega\right\}. \tag{5}$$

The distance throughput along s is composed of a visibility and transmittance term, $f(s) = V(s)T_\mathrm{r}(s)$, where $T_\mathrm{r}(s) = e^{-\sigma_t s}$ in homogeneous media. In the context of control variates, $f(s)$ becomes the integrand of interest, and the transmittance $T_\mathrm{r}(s)$ becomes the control variate $g(x)$. We expand the integrand and rearrange to obtain

$$\begin{aligned}
\langle D\rangle^{l-1,k}_{\text{Q-B1D}} &= C\int_{s_{k-}}^{s_{k+}} V(s)T_\mathrm{r}(s)\,\mathrm{d}s && (6)\\
&= C\int_{s_{k-}}^{s_{k+}} V(s)T_\mathrm{r}(s) - T_\mathrm{r}(s) + T_\mathrm{r}(s)\,\mathrm{d}s && (7)\\
&= C\left(\int_{s_{k-}}^{s_{k+}} T_\mathrm{r}(s)\,\mathrm{d}s + \int_{s_{k-}}^{s_{k+}}(V(s)-1)\cdot T_\mathrm{r}(s)\,\mathrm{d}s\right) && (8)\\
&= C\left(\frac{e^{-\sigma_t s_{k-}} - e^{-\sigma_t s_{k+}}}{\sigma_t} + \int_{s_{k-}}^{s_{k+}}(V(s)-1)\cdot T_\mathrm{r}(s)\,\mathrm{d}s\right) && (9)\\
&\approx C\left(\frac{e^{-\sigma_t s_{k-}} - e^{-\sigma_t s_{k+}}}{\sigma_t} + \frac{1}{N}\sum_{i=1}^{N}\frac{(V(s_i)-1)\cdot T_\mathrm{r}(s_i)}{p(s_i)}\right) && (10)
\end{aligned}$$

We can interpret the first term in Eq. (10) as the analytic integral of the contribution of the photon plane assuming full visibility. The second term is then a correction factor that accounts for occlusion along the camera segment.

In our implementation of short–short planes, we only take a single sample of the estimator in Eq. (10). We observe that in our test scenes, the vast majority of visibility tests (more than 95% on average) are unoccluded, and the estimate assuming full visibility is a good variate for the integrand. Fig. 1 demonstrates the variance reduction of control variates compared to naive Monte Carlo sampling of the blurred photon plane contribution.

1.1 Relation to Photon Beams

Our control variate estimator shares some similarities with prior work by Jarosz et al. [2011] on photon beams. Density estimation with Point×Beam (3D blur) involves a similar integration problem, and the analytic integral of the control variate in Eq. (10) is in fact identical to Eq. (11) in the paper by Jarosz et al. Unlike Point×Beam

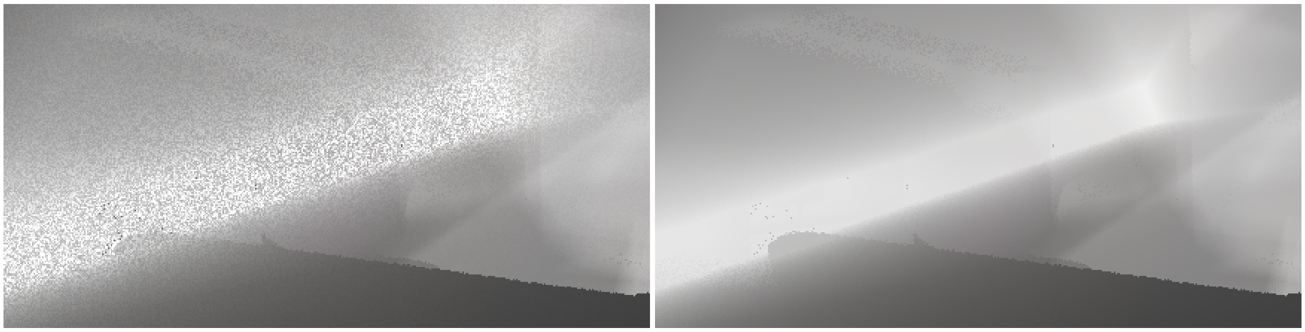

Fig. 1. We compare the variance of control variates (right) to naive Monte Carlo sampling (left) for integration through a camera-facing photon plane in the Living Room scene.

however, evaluating the contribution of a photon plane involves an additional visibility term, which explains the need for an additional Monte Carlo estimator in Eq. (10).

Although our implementation only supports photon planes, a similar control variate approach could be applied to rendering with photon volumes. However, the integrand in the photon volume case involves three transmittance terms that depend on the integration variable, and deriving an analytic expression for the control variate requires slightly more work. This shares some similarity with prior work on Beam×Beam (2D blur) estimators [Jarosz et al. 2011], which require evaluating integrals of two transmittance terms.

2 ANISOTROPIC SCATTERING

We show additional results of one of our estimators compared to photon beams in Fig. 2, for a medium exhibiting pronounced anisotropic scattering (mean scattering cosine of 0.9). With increasing anisotropy, photon scattering directions at subsequent bounces become increasingly colinear on average. This both decreases the average photon plane area, and increases the influence of the photon plane Jacobian. Nonetheless, photon planes still show significant variance reduction compared to photon beams at equal render time, even in a medium with high anisotropy.

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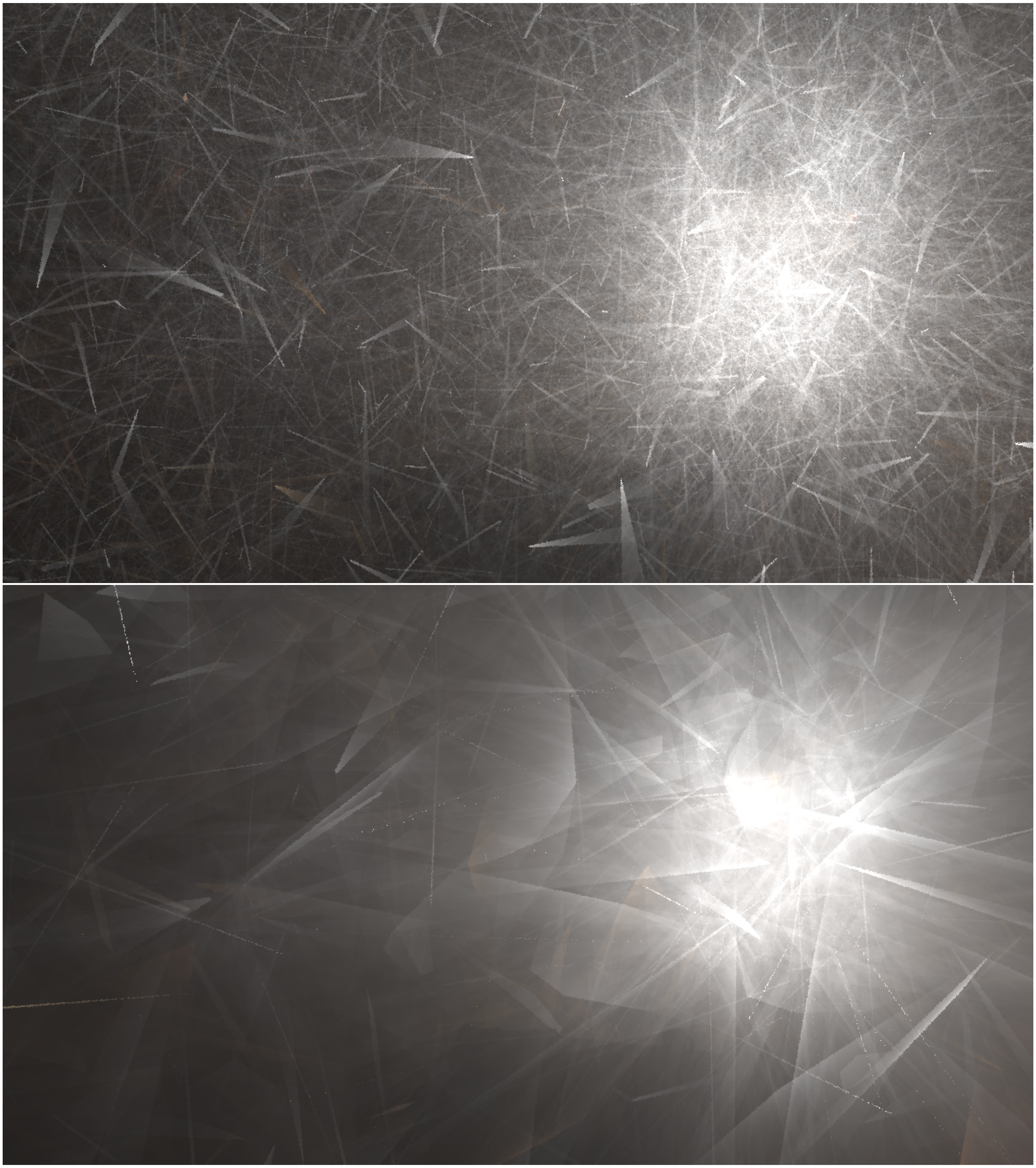

Fig. 2. We show equal-time renderings of photon beams (1D blur, top image) and unblurred photon planes (bottom image) in the KITCHEN scene after 10 minutes. The medium exhibits strong anisotropic scattering with a mean scattering cosine of 0.9.